Modeling the contact between a rolling sphere and a compliant ground plane

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Dec, 1, 2010

Introduction

Idea

Obtaining a reasonable approximation to a general robot foot by using a union of several spheres.

Objective

Simulating the motion of a rolling sphere on the ground.

Energy Audit

Also it is desirable for the model to support energy audit (a complete account of energy flows in the system).

Classical Models

- Hertz's theory for the contact force between a sphere and a plate.

$$F = kz^{\frac{3}{2}}$$

- A general non-linear equation with damping term was first introduced by Hunt and Crossley [1975].

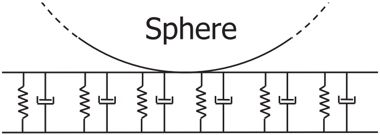
$$F = kz^{n} + \lambda z^{p} \dot{z}^{q}$$

- Hunt and Crossley [1975], Lankarani and Nikravesh [1990], and Marhefka and Orin [1999] have set the values of these parameters as $n = \frac{3}{2}$, $p = \frac{3}{2}$ and $q = 1$.

$$F = kz^{\frac{3}{2}} + \lambda z^{\frac{3}{2}} \dot{z}$$

Our Model

- The ground is considered to contain a uniform distribution of infinitely many non-linear spring-damper pairs.



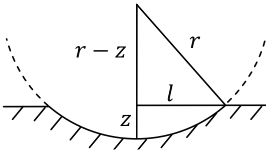
- According to our model, the normal force is given by:

$$f = f_K + f_D$$

where f_K is the summation of the spring forces and f_D is the summation of the damper forces.

Contact Area

- Defining the contact area to be the area of undeformed ground that makes contact with the sphere



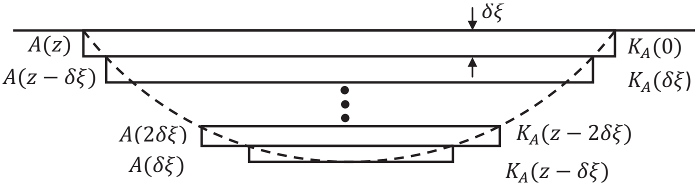
$$A(z) = \pi l^2 = \pi(2rz - z^2) = 2\pi rz(1 - \frac{z}{2r})$$

assumming $z \ll 2r$, for $z > 0$ we have

$$A(z) = 2\pi rz$$

Spring Force

- Defining $K_A(z)$ as a function of z.



$$f_K = \sum A(z)K_A(0)\delta\xi + A(z-\delta\xi)K_A(\delta\xi)\delta\xi + \ldots + A(\delta\xi)K_A(z-\delta\xi)\delta\xi$$

which gives us

$$f_K = \int_0^z A(\xi)\, K_A(z-\xi)\, \mathrm{d}\xi$$

Spring Force

- Choosing $K_A(z)$ in order to conform with Hertz's theory:

$$K_A(z) = \frac{E^*}{2\pi\sqrt{r}} z^{-\frac{1}{2}}$$

Where

$$\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}$$

So the spring force is

$$f_K = \left\{ \begin{array}{cc} \frac{4}{3} E^* \sqrt{r}\, z^{\frac{3}{2}} & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{array} \right\} = K_n z^{\frac{3}{2}}$$

Damper Force

- Damping force (f_D) is

$$f_D = \int_{A(z)} D_A(\zeta(A))\,\dot{z}\,\mathrm{d}A$$

- Assumming that $D_A(z)$ has the same relation with z as $K_A(z)$ does:

$$D_A(z) = \alpha\, z^{-\frac{1}{2}}$$

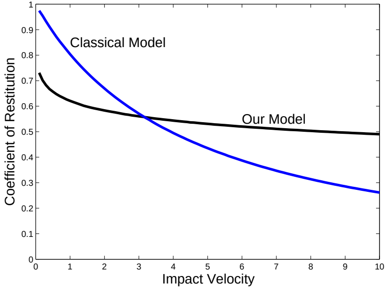
We have

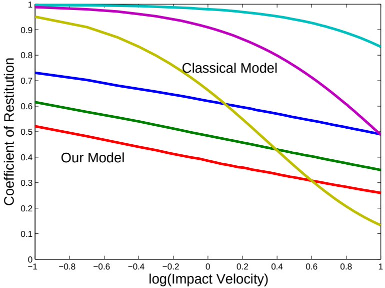
$$f_D = \left\{ \begin{array}{cc} 4\pi r \alpha z^{\frac{1}{2}}\dot{z} & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{array} \right\} = D_n z^{\frac{1}{2}}\dot{z}$$

Our Normal Force Model

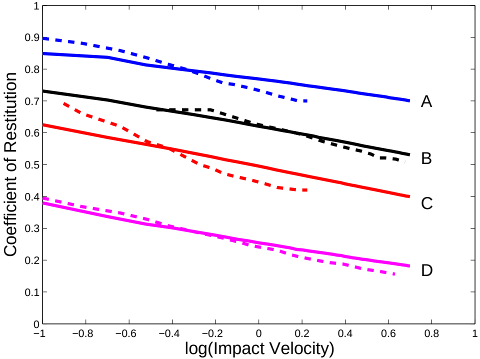
$$f = K_n z^{\frac{3}{2}} + D_n z^{\frac{1}{2}}\dot{z}$$

Coefficient of Restitution





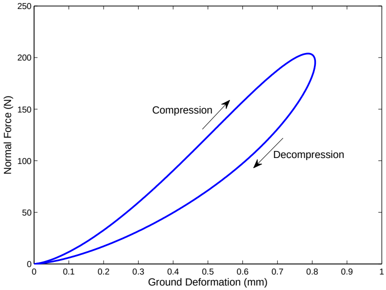
Coefficient of Restitution



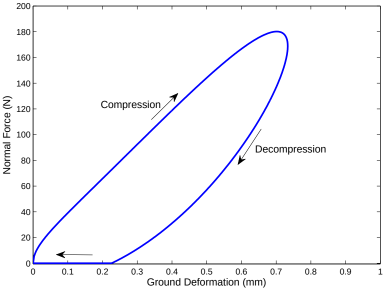
- A: a steel sphere ($r = 1.27$cm) and a cast iron plate
- B: a steel sphere ($r = 1.65$cm) and a cork plate
- C: a steel sphere ($r = 1.27$cm) and a brass plate
- D: a steel sphere ($r = 1.27$cm) and a cold-worked lead plate.

Bouncing Simulation

- In bouncing, our model predicts that the sphere will lose contact with the ground before z has fully returned to zero.



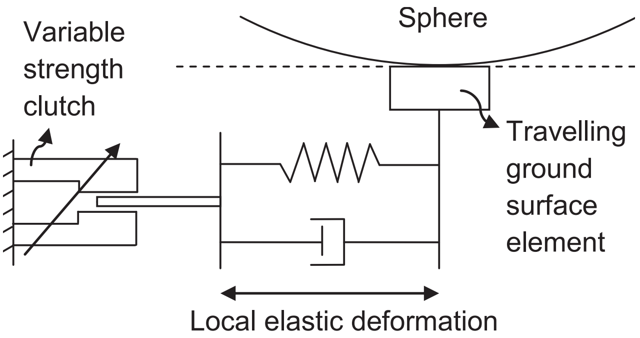
Classical Model



Our Model

Friction Model

- Our friction model consists of a spring, a damper and a clutch. The clutch is designed to slip when the ground reaction force reaches the edge of the friction cone.



Stiction and Slipping Forces

- Stiction force in each tangential direction is calculated by:

$$\mathbf{f}_{stick} = -k_t \mathbf{u} - b_t \mathbf{V}_{sph}$$

where

$$k_t = \int_{A(z)} K_A(\zeta(A))\,\mathrm{d}A \quad \text{and} \quad b_t = \int_{A(z)} D_A(\zeta(A))\,\mathrm{d}A$$

- By assumming that both surfaces are isotropic:

$$\mathbf{f}_{stick} = -K_t z^{\frac{1}{2}} \mathbf{u} - D_t z^{\frac{1}{2}} \mathbf{V}_{sph}$$

where

$$K_t = 2E^* \sqrt{r} \quad \text{and} \quad D_t = 4\pi r\,\alpha$$

- Slipping force would be calculated by:

$$\mathbf{f}_{slip} = \mathbf{f}_{stick} \times \frac{\mu F_n}{|\mathbf{f}_{stick}|} - C_V \mathbf{V}_{clutch}$$

Energy Audit

- Energy audit: keeping track of all energy in the system
- Energy conservation principle: at any instant of the simulation, the summation of the amount of energy which is dissipated and the amount of energy which is stored in springs and the body itself must be constant.

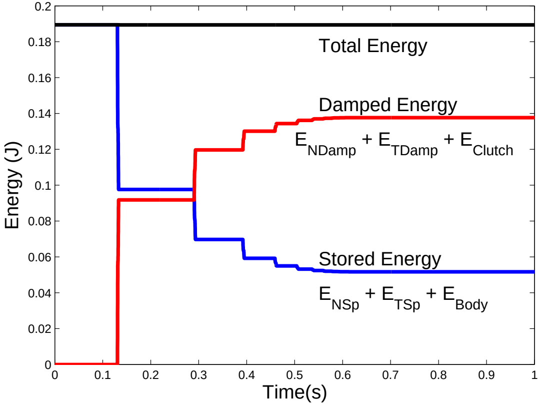
$$E_{Dissipated} + E_{Springs} + E_{Body} = \text{constant.}$$

Energy Audit

- Dissipated energy
  - normal damper (E_{NDamp})
  - tangential dampers (E_{TDamp})
  - clutch (E_{Clutch})
- Stored energy
  - normal spring (E_{NSp})
  - tangential springs (E_{TSp})

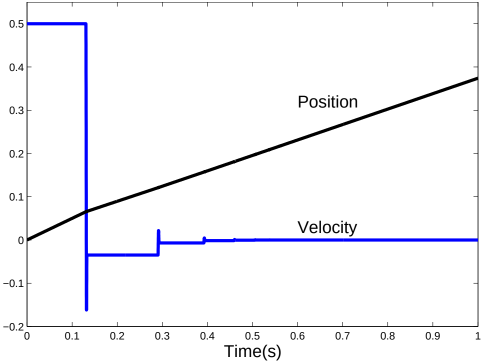
Energy Plot

- Sphere is released from 10cm height with velocity of $0.5\frac{m}{s}$ in both x and y directions.



Rolling Motion

- The x components of the position of the COM and the velocity of the contact point



Conclusions

- A complete full 3D nonlinear contact model which is able to calculate both normal and friction forces.
- A new nonlinear normal force model
- Predicting the values of the coefficient of restitution more accurately than the previous classical ones.
- A new nonlinear friction force model in two dimensions.
- Simulating the rolling motion of a sphere on the ground

Thank You Very Much!!

Q&A